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(54) Title: A METHOD FOR SOLVING FINITE ELEMENT MODELS USING TIME SLABBING

(57) Abstract: A method for solving finite element problems in n+1 dimensions by iteratively extruding an n-dimensional finite element mesh in an n+1th dimension to form "slabs" which can be more easily solved within the entire n+1-dimensional problem. In a preferred embodiment, a three-dimensional unstructured finite element mesh representing a physical system is extruded in the time dimension. The four-dimensional prisms formed by the extrusion are divided into simplices, forming the four-dimensional finite element mesh of an individual time slab. Time slabs corresponding to a series of time intervals are sequentially generated and solved. In a preferred embodiment, only a few time slabs are stored in working memory at a time so that a reduced amount of memory (in comparison to conventional methods of solving comparable problems) is required.

A METHOD FOR SOLVING FINITE ELEMENT MODELS USING TIME SLABBING

RELATED APPLICATIONS

This application claims priority under 35 U.S.C. § 119(e) to United States Patent Application No. 60/215,697 entitled "Method and System for Oil Reservoir Simulation and Modeling" by Stephen R. Kennon, Kok Thye Lim, Scott A. Canaan, Steven B. Ward, Stuart W. Pond, Jr. and Edward J. Barragy, filed June 29, 2000, which is incorporated by reference as if set forth in its entirety herein.

TECHNICAL FIELD OF THE INVENTION

This invention relates generally to methods for modeling physical systems using finite element analysis and, more specifically, to methods for creating finite element models used in simulating geological structures and corresponding fluid flow, especially in hydrocarbon reservoirs.

BACKGROUND OF THE INVENTION

Physical systems can be modeled mathematically to simulate their behavior under certain conditions. There are a wide variety of means to model these systems, ranging from the very simplistic to the extremely complicated. One of the more complicated means to model physical systems is through the use of finite element analysis. As the name implies, finite element analysis involves the representation of individual, finite elements of the physical system in a mathematical model and the solution of this model in the presence of a predetermined set of boundary conditions.

In finite element modeling, the region that is to be analyzed is broken up into sub-regions called elements. This process of dividing the region into sub-regions may be referred to as discretization or mesh generation. The region is represented by functions defined over each element. This generates a number of local functions that are much simpler than those which would be required to represent the entire region. The next step is to analyze the response for each element. This is accomplished by building a matrix that defines the properties of the various elements within the region and a vector that defines the forces acting on each element in the structure. Once all the element matrices and vectors have been created, they are combined into a structure matrix equation. This equation relates nodal responses for the entire structure to nodal forces. After applying boundary conditions, the structure matrix equation can be solved to obtain unknown nodal responses. Intra-element responses can be interpolated from nodal values using the functions which were defined over each element.

In prior art methods, when a solution to a structured three-dimensional model over time was desired, the model had be solved for a first time value, then again for a second time value, and so on, until a series of time steps covering the desired range was completed. If it was desired to model a temporal feature more accurately, the size of the time steps had to be decreased and the entire three-dimensional physical model had to be solved for a greater number of time values. The computing resources which were required to generate the solution obviously increased with the number of time steps for which the model was solved. There was no way to localize the smaller time

steps to a particular physical volume of interest or to adapt the three-dimensional spatial model over time. Further, when an error occurred in the solution of the model for one of the time values, the model and/or the time steps had to be adjusted and the entire model solved again for each of the time values.

SUMMARY OF THE INVENTION

One or more of the problems outlined above may be solved by the various embodiments of the present invention. Broadly speaking, the invention comprises a method for solving space-time problems involving three-dimensional space by constructing a three-dimensional finite element model, then repeatedly extruding the model in a temporal dimension to form a "time slab" and solving the slab. This method may provide a number of advantages over prior art methods, including decreased memory usage, the ability to refine the spatial mesh from one time slab to another, the ability to adapt the temporal mesh to achieve convergence of a solution for the slab, the ability to perform "checkpointing" of solutions for successive time slabs, and so on.

In one embodiment, the method comprises providing a three-dimensional mesh which models the system to be solved and extruding the three-dimensional mesh in a series of time slabs which are sequentially solved. Each slab is visited, solved, adapted and re-solved if necessary to meet any error criteria, and then archived. Each slab includes a plurality of nodes that represent a region of four-dimensional space (i.e., space-time.) As each slab is solved, it may be desirable to refine the slab in the particular spatial regions of interest (for example, to

obtain more accurate solutions in these regions.) Likewise, it may be desirable to refine the slab in the temporal dimension (for instance, to achieve convergence of the solution for the slab.) The solution for each slab may therefore be examined to determine whether refinement of the slab is necessary and, if so, the slab is adapted spatially (new nodes are added) and/or temporally (time steps are shortened.) After the slab has been appropriately refined, a new solution for the slab is generated. Because the solutions of the previous time slabs were completed and archived, there is no need to re-solve those slabs.

In one embodiment, all of the slabs to be solved are generated prior to the solution of any one of the slabs. In another embodiment, a first slab is generated and solved before a second slab is generated. The slab may be refined and re-solved as necessary to obtain a valid solution. After this solution is obtained, the slab is extruded to form a subsequent slab. (Although each slab is extruded from the spatial mesh which forms the boundary between the slabs, this may be more conveniently referred to as extruding one slab from the previous slab.) This subsequent slab is then solved, and the process is repeated until the desired time range has been covered. In another embodiment, the current time slab and at least one subsequent slab are generated. The current slab is then solved, including any refinements which are necessary. After the current slab has been solved, a new slab is extruded from the subsequent slab rather than the current slab. Because any particular slab is generated from the previous slab before the previous slab is refined (during the process of solving it,) refinements to each slab which

do not need to be carried through from one slab to the next are not propagated through the slabs.

Numerous alternative embodiments are also possible. Particularly, it should be noted that the present methods, though described primarily with reference to the modeling of three-dimensional physical systems (e.g., oil reservoirs,) may be applicable in other contexts as well.

BRIEF DESCRIPTION OF THE DRAWINGS

Other objects and advantages of the invention may become apparent upon reading the following detailed description and upon reference to the accompanying drawings in which:

FIGURE 1 is a flow diagram illustrating the basic steps of one embodiment of the present invention;

FIGURE 2 is an illustration of the triangular and tetrahedral elements which comprise unstructured finite element meshes in one embodiment of the present invention;

FIGURE 3 is a flow diagram illustrating a method according to an alternative embodiment of the present invention;

FIGURE 4 is a flow diagram illustrating a method according to an alternative embodiment of the present invention;

FIGURE 5 is a diagram illustrating the relationship of the time slabs in one embodiment of the present invention;

FIGURES 6A-6F are a series of diagrams showing the manner in which increased accuracy can be achieved in prior art methods and in one embodiment of the present invention;

FIGURE 7 is a diagram illustrating the propagation of refinements to subsequent time slabs; and

FIGURE 8 is a diagram illustrating the manner in which generating time slabs which follow a current time slab may limit propagation of refinements to the subsequent time slabs.

While the invention is subject to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and the accompanying detailed description. It should be understood, however, that the drawings and detailed description are not intended to limit the invention to the particular embodiment which is described. This disclosure is instead intended to cover all modifications, equivalents and alternatives falling within the spirit and scope of the present invention as defined by the appended claims.

DETAILED DESCRIPTION OF A PREFERRED EMBODIMENT

A preferred embodiment of the invention is described below. It should be noted that this and any other embodiments described below are exemplary and are intended to be illustrative of the invention rather than limiting.

The present method is particularly well-suited to solving space-time problems involving three-dimensional space. In the prior art, three-dimensional finite element models were solved for a particular point in time. Put another way, the models were three-dimensional instead of four-dimensional. If a solution for the three-dimensional model over time was required, the three-dimensional model was solved for a first time value, then for a second time

value, then a third, and so on. Thus, the finite element model was not a four-dimensional model, but was instead a three-dimensional model which was stepped over a given range of time values. The series of three-dimensional solutions to the three-dimensional model could then be combined to construct a four-dimensional solution.

A preferred embodiment of the present method employs a series of four-dimensional finite element models to generate a four-dimensional solution. More particularly, an unstructured finite element mesh is generated in the form of a series of time slabs which model the desired physical system over a desired time span. Referring to FIGURE 1, a flow diagram illustrating the basic steps of one embodiment of the present method is shown. As shown in the figure, the method comprises generating a plurality of time slabs and solving them. Each of the time slabs is sequentially visited and solved to generate a solution for that slab. This process is continued until all of the time slabs have been solved. The combined solutions for all of the slabs comprise a solution for the three-dimensional system over time. The unstructured four-dimensional finite element model is capable of following irregular spatial and temporal features of the modeled system, and the solution of the successive time slabs greatly reduces the memory required to generate a solution.

The details of a preferred embodiment will be set forth below. It may be helpful, however, to first define a few terms.

A node is a point in space. In finite element modeling, nodes form the vertices of the elements which are modeled. The nodes also form part of a mesh of nodes and edges which

define the boundaries between elements in the modeled space.

An edge is a line between two nodes which form vertices of an element. The edges form part of the mesh which defines the boundaries between elements in the modeled space.

A simplex is a spatial configuration of n dimensions determined by n + 1 points in a space of dimension equal to or greater than n. In other words, a—simplex is a geometric spatial element having the minimum number of boundary points necessary to enclose a space in a given number of dimensions. For example, in two dimensions, a simplex comprises a triangle, together with the interior area bounded by the triangle (see FIGURE 2.) Two points are insufficient to form a simplex in two-dimensional space because no area is bounded by the points (and the lines which interconnect them.) While four points may be sufficient to bound a two-dimensional area, they do not comprise the minimum number of boundary points by which the two-dimensional area can be bounded. In three dimensions, a simplex comprises a tetrahedron, which is bounded by four vertices (see FIGURE 2.) In four dimensions, a simplex comprises a hypertetrahedron (sometimes referred to as a hypertet) having five vertices.

A mesh is a collection of elements that fill a space. These elements are representative of a system which resides in that space. Because each element can be defined by a plurality of nodes and/or the edges between those nodes, a mesh may alternatively be considered a collection of nodes and/or the edges between them. At various points in this disclosure, "mesh" will be used to alternately refer to collections of elements or nodes/edges, depending upon the

context in which the term is used. The mesh may also be referred to herein as a finite element model or simply a model.

Referring to FIGURE 3, a flow diagram illustrating a method according to an alternative embodiment of the method is shown. The four-dimensional mesh of the time slabs in the preferred embodiment is generated from a three-dimensional mesh by extruding each of the simplices of the three-dimensional mesh in a fourth dimension (time.) The extrusion creates a four-dimensional prism from each of the three-dimensional simplices. These four-dimensional prisms are then divided into a plurality of four-dimensional simplices which form a four-dimensional finite element model. This process is shown graphically in FIGURE 4. The extrusion can be repeated to form a slab that extends over a predetermined time range (although it should be noted that a slab can consist of a single extruded layer.) The elements of this model can be selectively refined to obtain a finer mesh in areas of greater interest, and a coarser mesh in areas which are of less interest. The mesh can be refined in the spatial dimensions (x, y, z) and, unlike time-stepped models, in the temporal dimension as well.

The time slabs are solved sequentially. In other words, a first one of the slabs is solved, then the next slab is solved, then the next, and so on. In one embodiment, after each of the time slabs is solved, it is archived. Because of the causal nature of time, the assumption can be made that no data relating to earlier time slabs will change once those slabs are solved. If a solution to a particular time slab does not converge, or if the solution does not meet the error criteria, the solution can be discarded, the

mesh refined, and the time slab re-solved. It is not necessary to re-solve any of the earlier time slabs because the refinement and re-solving of the current slab does not affect the earlier slabs. (In conventional methods, the same three-dimensional mesh is time-stepped over the desired range, so if changes in the mesh are necessary at a later time step, they will have to be present at the earlier time steps as well. Thus, such changes necessitate that the entire solution be discarded.)

Referring to FIGURE 5, a diagram illustrating the relationship of the time slabs in one embodiment of the present method is shown. In this figure, the time slabs which correspond to earlier times are lower and the time slabs which correspond to later times are higher. It is assumed in this figure that each time slab is generated, then solved, then archived. (Other embodiments may maintain more time slabs in memory or may generate a subsequent time slab before solving the current one.) Three adjacent time slabs are depicted in FIGURE 5. The earliest time slab 21 has been generated, solved and archived, so it is no longer in memory. Time slab 22 is the current slab. Time slab 22 has been generated and is being solved. This is the only time slab in memory in this embodiment. Time slab 23 has not yet been generated, and is therefore shown as a dashed outline. Time slab 23 will be generated after current time slab 22 has been solved and archived.

The present method has memory requirements which may be substantially less than conventional methods, depending upon the particular implementation. In one embodiment, all of the time slabs are stored in core memory while

successive slabs are solved. This embodiment may provide only a minor improvement over the prior art in terms of memory usage. In another embodiment, only the slab for which a solution is currently being computed is retained in core memory -- earlier slabs are archived since they are not needed for the current solution and will not be changed. In this case, the memory requirements are substantially reduced from the prior art. Still other embodiments may store more than one, but less than all of the time slabs in memory, thereby providing different magnitudes of reductions in memory usage.

In a preferred embodiment of the present method, the three-dimensional mesh representing the system to be modeled is preferably an unstructured mesh of three-dimensional simplices. (While some embodiments may be implemented using structured meshes, this disclosure will focus primarily on those embodiments which use unstructured meshes.) An unstructured mesh is preferred because the elements of unstructured meshes are not constrained by the same requirements as structured meshes (e.g., minimum size of elements) and are therefore capable of more closely matching nodes of the elements to points of interest (e.g., object surfaces or temporal disturbances.) As a result, unstructured meshes are typically capable of more accurately modeling physical systems than comparable structured meshes.

The three-dimensional mesh is extruded in the temporal dimension to form a time slab. The extrusion which forms the time slab extends generally from a first time to a second time, covering an interval which is a portion of the entire range of time over which a solution is desired. The

time slabs may be said to generally correspond to a time interval because the nodes of the mesh may not correspond to only the "beginning" and "ending" times of the interval. The nodes are part of a four-dimensional mesh and may be adjusted temporally, as well as spatially, to more accurately reflect temporal and spatial features of the modeled system. Consequently, a particular time slab may not extend in a well-defined manner from a certain beginning time to a certain end time.

As indicated above, prior art methods do not actually solve a four-dimensional model. Instead, a three-dimensional model is solved at each of a plurality of time steps. Thus, if increased accuracy is desired in a particular time interval, the entire three-dimensional spatial model may be solved for an increased number of time values (using smaller time steps.) The structured mesh cannot be refined to have a first time step in a in a first spatial region and at the same time have a different time step in a second spatial region. Using the unstructured mesh of the present system and method, however, this is easily achieved.

This is illustrated in FIGURES 6A-6F. FIGURES 6A-6B comprise a series of diagrams showing the manner in which increased accuracy can be achieved in prior art methods and in one embodiment of the present method. FIGURE 6A depicts a one-dimensional mesh 30 comprising nodes 31-33. Mesh 30 is stepped through times n, n+1 and n+2 to generate a solution over time. In other words, a solution is generated at time n, then another solution is generated at time n+1, and finally a solution is generated at time n+2. In comparison, FIGURE 6B depicts a one-dimensional mesh 30 which is extruded to form a two-dimensional mesh. This

single mesh includes nodes 31-33 and 41-43. A second two-dimensional mesh is defined by nodes 41-46. Each of these two-dimensional meshes comprises a time slab extending from time n to time n+1, or from time n+1 to time n+2. (In an alternative embodiment, nodes 31-33 and 41-46 could define a single time slab extending from time n to time n+2.)

If a solution for the model varies rapidly with time, it may be desirable to modify the model to obtain a higher level of accuracy in the solution. In the case of the prior art method, this may be accomplished by reducing the time steps at which solutions for the one-dimensional model are computed, as illustrated in FIGURE 6C. As shown in this figure, solutions for the one-dimensional mesh are calculated at times n (30), n+0.5, (34), n+1.0, (35), n+1.5 (36) and n+2.0 (37). It can be seen from the figure that whenever time step is selected, this same step is applied over the extent of the one-dimensional model 30 -- that time step is not varied from one part of the model to another. Consequently, if the rapid changes are confined to only a portion of the model, computational power may have to be wasted on computations involving unnecessary nodes. Using the present methods, the time slabs can be modified to add nodes only in the region of interest. Referring to FIGURE 6D four-dimensional, the one-dimensional model is extruded to form to time slabs as shown in FIGURE 6B, but then two additional nodes, 51 and 52, are added in the region which exhibits the rapid change (at the center of each slab.) Thus, computations involving times n+0.5 and n+1.5 involve only a single node each.

Similarly, if the solution for the model changes rapidly as a function of spatial position, but does so only at certain

times, it may be possible to increase the accuracy of the solution for the model over time. As depicted in FIGURE 6E, this can be accomplished in prior art methods by increasing the number of nodes in the one-dimensional mesh. In this figure, new nodes 38 and 39 have been added. This mesh is solved at each time step, whether the extra nodes are needed at that particular time or not. Using the present methods, however, the computational requirements can be decreased by extruding the original one-dimensional model to form the time slabs, then adding new nodes as necessary to increase accuracy in the time and region of interest. As shown in FIGURE 6F, nodes 53 and 54 are added at time n+1. Consequently, the computational requirements associated with time n+1 are increased, but the computational requirements associated with times n and n+2 remain the same.

While the examples of FIGURES 6A-6F illustrate the case of a one-dimensional mesh which is solved over time, it should be noted that these examples can be extended to two, three, or even more dimensions (although the three-dimensional case is expected to be most prevalent.)

The usefulness of this capability can be illustrated using the following example. If a disturbance such as a pressure wave propagates through a reservoir, it may be desirable to improve the accuracy of the model by increasing the number of elements in the vicinity of the disturbance. Because the disturbance propagates through the reservoir as a function of time, the location at which the fine mesh is needed changes with time. In this instance, it is assumed that the disturbance will propagate through the entire space which is being modeled, so, at one time or another,

each location in the modeled space will need to have an increased number of elements.

A structured three-dimensional mesh does not change with time. It is simply time-stepped, with each element in the mesh being advanced by the same interval (size of time step.) Thus, each region in the modeled space must have the maximum number of elements that will be desired at any time. A solution for the model at any given time requires solutions for all of these elements, regardless of whether the additional elements are needed at that particular time. As a result of the increased number of elements in the mesh, increased computing resources are required to generate a solution.

The present methods solve this problem by generating a space-time mesh that can be refined for particular regions of both space and time. While the n-dimensional mesh in the prior art is not altered from one time step to the next, the n+1-dimensional mesh used in the present method is generated slab-by-slab, with each slab being refined as needed to obtain the desired level of accuracy in particular areas. If this level of accuracy is not needed in later slabs, those slabs can be refined to consolidate elements in the mesh and thereby reduce the computational requirements of those slabs. Another advantage that may be provided by the present methods is that the nodes which define a particular slab need not all have the same time value. While prior art methods take all of the nodes of the mesh at a single point in time and calculate a solution, the present methods may calculate a solution for a set of nodes that have different time values. This may

be useful, for example, to model a non-planar (e.g., spherical) wavefront passing through the modeled system.

It should be noted that the time slabs used in the present methods may comprise a single layer of elements, or multiple layers. A time slab which has a single layer may be generated by a single extrusion, resulting in each element having nodes on both sides of the slab. A time slab which has multiple layers may be generated by extruding an initial mesh in the temporal dimension, subdividing the resulting prisms into simplices, and repeating this process at least one more time to create additional layers of elements. In a multi-layer time slab, none of the elements will have nodes which lie on both faces of the slab, unless the slab is refined by collapsing together two or more elements which have nodes on both faces.

Generally speaking, the method comprises the successive generation and solution of n+1-dimensional finite element meshes, where each mesh extends generally over a corresponding interval of time (i.e., a time slab.) Because each time slab can be solved separately, the computational requirements for generation of a solution are simplified, and the memory requirements may be substantially reduced. The different embodiments may generate either structured or unstructured meshes (although an unstructured mesh is preferred) of two, three, four, or even more dimensions. The time slabs may be generated immediately prior to being solved, they may be generated one or two slabs ahead of the current slab (to limit the propagation of refinements,) or they may all be generated at once. The different embodiments may retain only the

current time slab in memory, or they may retain several (e.g., the current slab and one or two succeeding slabs) or even all of the slabs in memory. Different embodiments may refine the time slabs to add nodes or collapse elements, or they may not refine the slabs at all.

As indicated above, the preferred embodiment generates four-dimensional time slabs from a three-dimensional finite element mesh. The initial three-dimensional mesh, as well as the four-dimensional time slabs comprise unstructured meshes. The generation of the four-dimensional mesh can be performed in a number of ways. A description of a preferred method follows. First, an unstructured three-dimensional mesh which is representative of the physical system to be modeled (i.e., a mesh in X-Y-Z space) is generated. This three-dimensional mesh may be generated by any suitable means. This three-dimensional mesh is then extruded in the temporal dimension. (A preferred method of extrusion of the three-dimensional mesh will be described in more detail below.) The three-dimensional mesh is extruded over a certain interval which is equivalent to the minimum time step that would be desired at any region in the three-dimensional mesh during that interval. The extrusion creates a plurality of four-dimensional prisms, each of which is then broken up into three four-dimensional simplices, creating an unstructured four-dimensional mesh extending over the interval.

The extrusion and creation of simplices is then repeated for a succeeding interval. (It should be noted that each successive extrusion is made from the three-dimensional mesh displaced in the previous extrusion, and not from the four-dimensional simplices.) The four-dimensional mesh of

the first interval is then examined to identify regions in which the mesh should be refined (i.e., nodes added and corresponding new simplices defined, or nodes eliminated and simplices collapsed.) In one instance, regions in which the minimum time step is not necessary may identified and elements in the region collapsed into fewer elements having larger time steps. This may result in an unstructured four-dimensional mesh which has varying time steps in different regions of the modeled four-dimensional space. (In an alternative embodiment, it is possible to generate a mesh that uses larger time steps and then add nodes in regions of interest, wherein the new nodes are at times which are intermediate to the original time steps.) In another instance, regions in which additional information is needed may be identified so that additional nodes can be inserted into the mesh and corresponding new elements defined.

As indicated above, this embodiment involves the generation of not only the time slab which will next be solved (the "current" time slab,) but also a succeeding time slab. By generating the succeeding time slab prior to refining the current time slab, refinements which are made to the current time slab will not propagate through the subsequent time slabs. This is illustrated in FIGURES 7 and 8. FIGURE 7 is a diagram illustrating the propagation of refinements to subsequent time slabs. In this figure, a one-dimensional mesh 60 is extruded over the interval t0 to t1 to form a first time slab 61. (It can be seen that the time slab in this instance comprises two layers.) Current time slab 61 is then refined. In this case, the refinement consists of the addition of two new nodes 63 and 64. After current time slab 61 is solved, a succeeding time slab 62

is generated by extruding the refined one-dimensional mesh at time t1. Succeeding time slab 62 therefore includes nodes 65-68, whether they are necessary or not, simply because they were extruded from nodes 63 and 64.

In the preferred embodiment, the second time slab is extruded over the interval t1 to t2 before refinements to the current time slab are made. This situation is illustrated in FIGURE 8. The current time slab 61 is again generated by extruding one-dimensional mesh 60 over the interval t0 to t1. The succeeding time slab 72 is extruded from the same one-dimensional mesh prior to making any refinements. After succeeding time slab 72 is generated, current time slab 61 is refined. Nodes 63 and 64 are again added to the current time slab. Because they form part of the boundary between current time slab 61 and succeeding time slab 72, these nodes are also part of the succeeding time slab. With a succeeding slab must therefore be refined to incorporate these nodes. While they are integrated into succeeding time slab 72, however, the additional nodes do not propagate through to the other side of the succeeding time slab. Consequently, they will not be included in the extrusion of the one-dimensional mesh to form a time slab from t2 to t3. They may nevertheless be added to succeeding time slab 72, if necessary, when that time slab is refined.

The benefits and advantages which may be provided by the present invention have been described above with regard to specific embodiments. These benefits and advantages, and any elements or limitations that may cause them to occur or to become more pronounced are not to be construed as a critical, required, or essential features of any or all of

the claims. As used herein, the terms "comprises," "comprising," or any other variations thereof, are intended to be interpreted as non-exclusively including the elements or limitations which follow those terms. Accordingly, a process, method, article, or apparatus that comprises a list of elements does not include only those elements but may include other elements not expressly listed or inherent to the claimed process, method, article, or apparatus.

While the present invention has been described with reference to particular embodiments, it should be understood that the embodiments are illustrative and that the scope of the invention is not limited to these embodiments. Many variations, modifications, additions and improvements to the embodiments described above are possible. Particularly, these variations may include computers or other data processing devices, computer readable media (such as floppy disks, CD-ROMs, DVD-ROMs, etc.,) storage devices, computer memories and the like which contain software, firmware or other programming embodying the foregoing methods. It is contemplated that these variations, modifications, additions and improvements fall within the scope of the invention as detailed within the following claims.

CLAIMS

1. A method for solving an n-dimensional finite element model over time, comprising:

generating a plurality of time slabs, wherein each time slab comprises an n+1-dimensional finite element mesh based on the n-dimensional finite element model; and

sequentially generating solutions for each of the plurality of time slabs.

2. The method of claim 1 wherein each n+1-dimensional finite element mesh corresponds generally to a time interval within a selected time range.

3. The method of claim 1 wherein a first time slab is extruded from the n-dimensional finite element model, and wherein each succeeding time slab is extruded from the preceding time slab.

4. The method of claim 3 wherein each succeeding time slab is extruded from the preceding time slab after the solution has been generated for the preceding time slab.

5. The method of claim 4 wherein each succeeding time slab is extruded from the preceding time slab before a solution has been generated for the preceding time slab.

6. The method of claim 1 further comprising refining each time slab prior to generating the solution for the time slab.

7. The method of claim 6 wherein refining each time slab comprises incorporating into a current time slab nodes which were added to a preceding time slab in a refinement of the preceding time slab.

8. The method of claim 6 wherein refining each time slab comprises incorporating one or more new nodes into the time slab and defining one or more corresponding new elements in the time slab.

9. The method of claim 6 wherein refining each time slab comprises removing one or more nodes from the time slab and collapsing one or more corresponding elements in the time slab.

10. The method of claim 6 wherein refining each time slab comprises defining a plurality of distinct time steps between a plurality of pairs of nodes in at least one of the plurality of time slabs.

11. The method of claim 1 further comprising archiving the solution for each time slab after the solution is generated.

12. The method of claim 11 further comprising maintaining only a current one of the plurality of time slabs in memory while a solution is generated for the current one of the plurality of time slabs.

13. The method of claim 1 further comprising maintaining each of the plurality of time slabs in memory while solutions are generated for each of the plurality of time slabs.

14. The method of claim 1 wherein generating solutions for each of the plurality of time slabs comprises, for at least one of the plurality of time slabs, computing a non-converging solution for the at least one time slab, refining the at least one time slab, and computing a new solution for the at least one time slab.

15. The method of claim 1 wherein generating solutions for each of the plurality of time slabs comprises, for at least one of the plurality of time slabs, computing a first solution for the at least one time slab, determining that the first solution is not within an error tolerance, refining the at least one time slab, and computing a new solution for the at least one time slab.

16. The method of claim 1 wherein generating a plurality of time slabs comprises generating each time slab as an n+1-dimensional unstructured finite element mesh.

17. A method for solving an n-dimensional finite element model over time, comprising:

generating a plurality of time slabs, wherein each time slab comprises an n+1-dimensional finite element mesh based on the n-dimensional finite element model and wherein each n+1-dimensional finite element mesh corresponds generally to a time interval within a selected time range;

sequentially generating solutions for each of the plurality of time slabs;

wherein a first time slab is extruded from the n-dimensional finite element model, and wherein each

succeeding time slab is extruded from the preceding time slab.

18. The method of claim 17 wherein each time slab comprises an n+1-dimensional unstructured finite element mesh.

19. The method of claim 17 wherein generating solutions for each of the plurality of time slabs comprises, for at least one of the plurality of time slabs, computing a first solution for the at least one time slab, determining that the first solution does not satisfy one or more error requirements, refining the at least one time slab, and computing a new solution for the at least one time slab.

20. The method of claim 17 further comprising maintaining only a current one of the plurality of time slabs in memory while a solution is generated for the current one of the plurality of time slabs and archiving the current one of the plurality of time slabs after the corresponding solution is generated.

21. A computer-readable medium which contains instructions configured to cause a data processing system to perform the method comprising:

generating a plurality of time slabs, wherein each time slab comprises an n+1-dimensional finite element mesh based on an n-dimensional finite element mesh; and

sequentially generating solutions for each of the plurality of time slabs.

22. The computer-readable medium of claim 21 wherein each n+1-dimensional finite element mesh corresponds generally to a time interval within a selected time range.

23. The computer-readable medium of claim 22 wherein a first time slab is extruded from the n-dimensional finite element model, and wherein each succeeding time slab is extruded from the preceding time slab.

24. The computer-readable medium of claim 23 wherein each succeeding time slab is extruded from the preceding time slab after the solution has been generated for the preceding time slab.

25. The computer-readable medium of claim 23 wherein each succeeding time slab is extruded from the preceding time slab before a solution has been generated for the preceding time slab.

26. The computer-readable medium of claim 22 further comprising refining each time slab prior to generating the solution for the time slab.

27. The computer-readable medium of claim 22 further comprising maintaining only a current one of the plurality of time slabs in memory while a solution is generated for the current one of the plurality of time slabs and archiving the current one of the plurality of time slabs after the corresponding solution is generated.

28. The computer-readable medium of claim 22 wherein generating solutions for each of the plurality of time slabs comprises, for at least one of the plurality of time slabs, computing a first solution for the at least one time slab, determining that the first solution is not within an

error tolerance, refining the at least one time slab, and computing a new solution for the at least one time slab.

29. The computer-readable medium of claim 22 wherein generating a plurality of time slabs comprises generating each time slab as an n+1-dimensional unstructured finite element mesh.

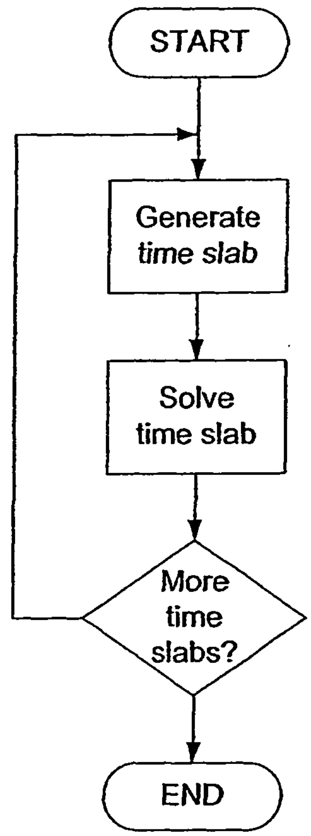
START
Generate
time slab
Solve
time slab
More
time
slabs?
END

Fig. 1

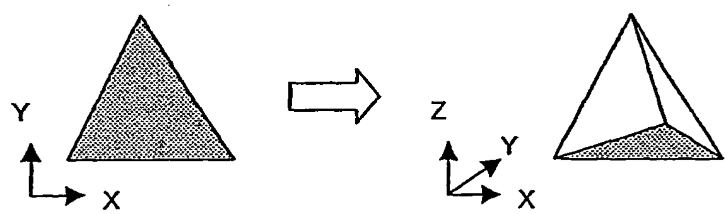

Fig. 2

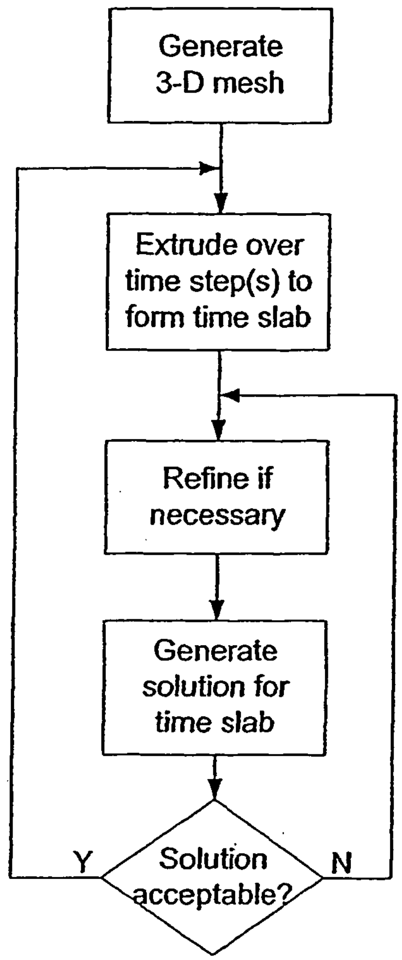
Generate 3-D mesh
Extrude over time step(s) to form time slab
Refine if necessary
Generate solution for time slab
Solution acceptable?
Y
N

Fig. 3

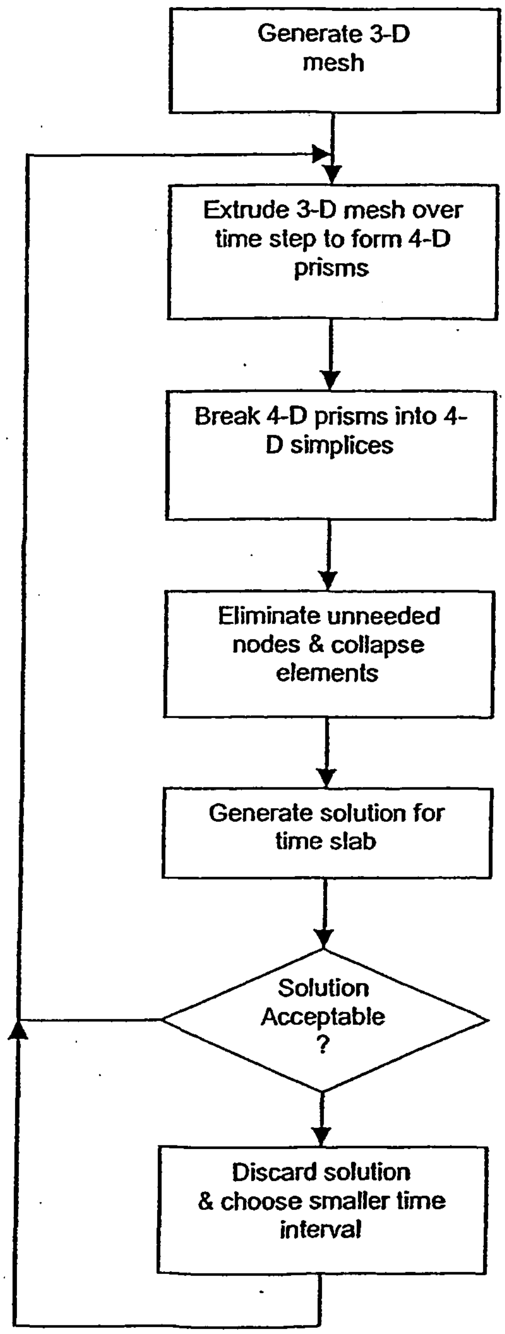
Generate 3-D mesh
Extrude 3-D mesh over time step to form 4-D prisms
Break 4-D prisms into 4-D simplices
Eliminate unneeded nodes & collapse elements
Generate solution for time slab
Solution Acceptable ?
Discard solution & choose smaller time interval

Fig. 4

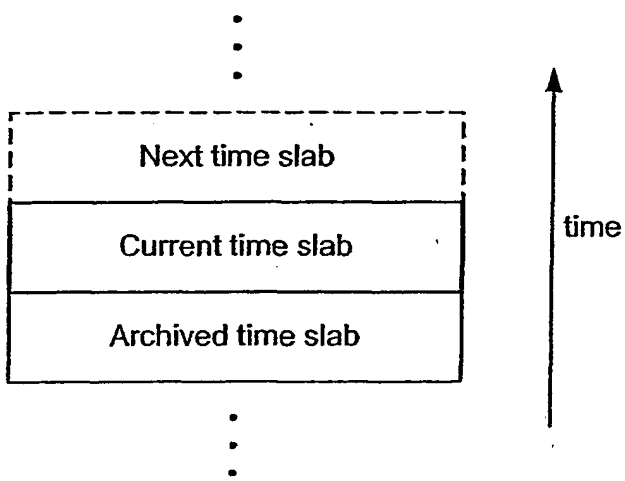

Fig. 5

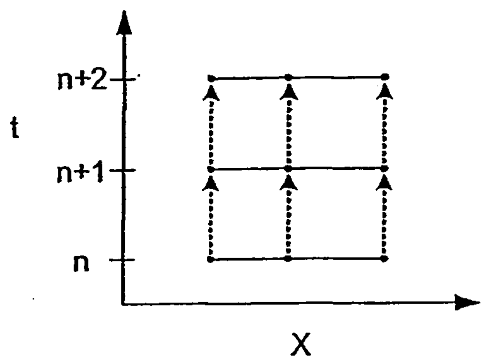

Fig. 6A

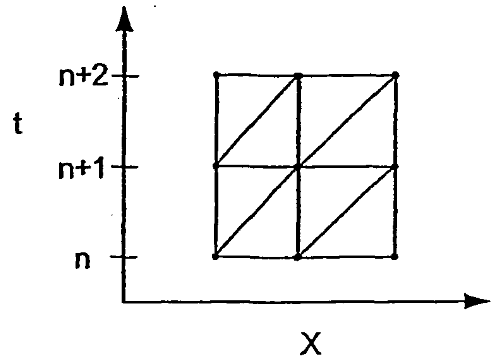

Fig. 6B

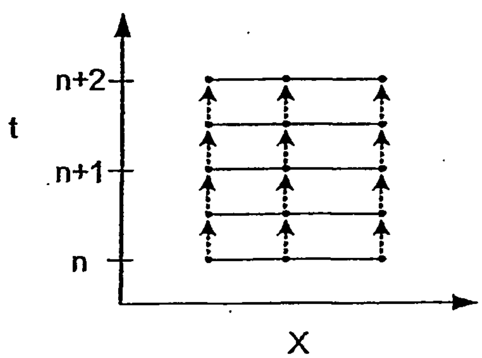

Fig. 6C

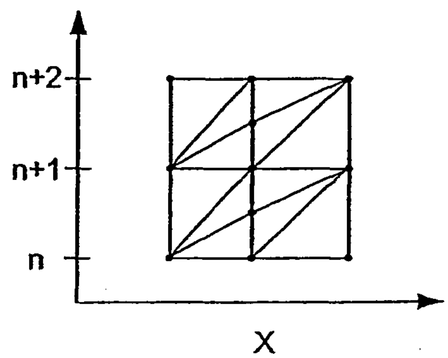

Fig. 6D

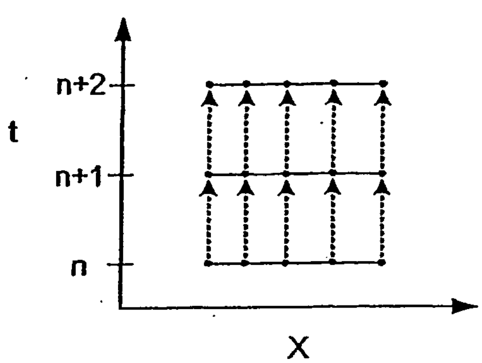

Fig. 6E

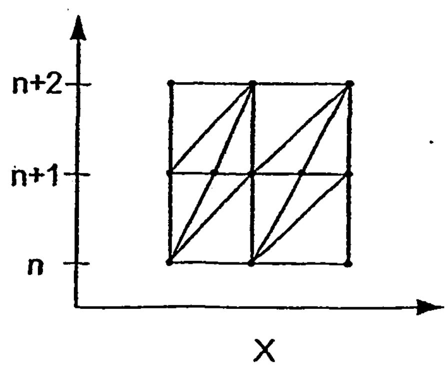

Fig. 6F

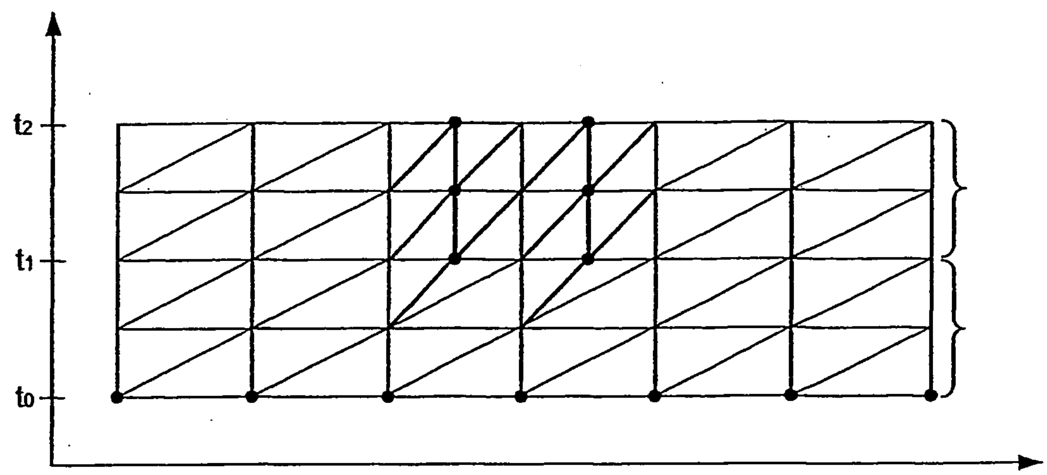

Fig. 7

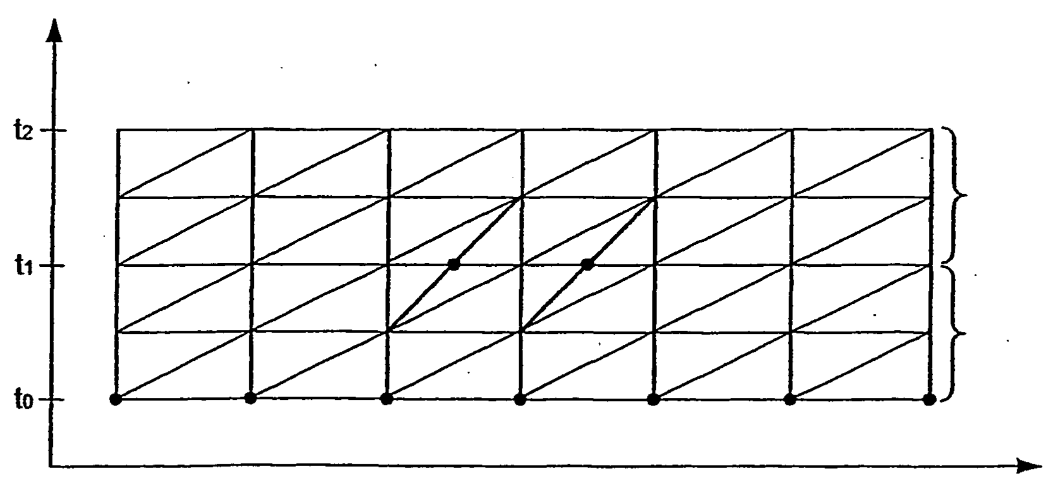

Fig. 8

INTERNATIONAL SEARCH REPORT

International Application No
PCT/US 01/20905

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 G01V11/00 G06F17/50

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 G01V G06F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 6 064 810 A (PRICE DONALD C ET AL) 16 May 2000 (2000-05-16) column 5, line 25 -column 7, line 42 column 15, line 30 - line 35; figure 9	1-4, 6-10,15, 17,19, 21-24,26
A	US 5 255 212 A (YAMAGUCHI SATAROU ET AL) 19 October 1993 (1993-10-19) column 1, line 41 -column 2, line 12 column 3, line 30 -column 5, line 2	1,17,21
A	EP 0 709 789 A (DORYOKURO KAKUNENRYO) 1 May 1996 (1996-05-01) page 5, column 8, line 4 -page 6, column 9, line 53	1,17,21

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[X] Further documents are listed in the continuation of box C.

[X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier document but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
25 February 2002	05/03/2002

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL – 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Lorne, B

2

International Application No
PCT/US 01/20905

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
A	WO 99 40532 A (EXXON PRODUCTION RESEARCH CO) 12 August 1999 (1999-08-12) page 3, line 8 - line 24 page 10, line 28 -page 11, line 31	1,17,21

2

INTERNATIONAL SEARCH REPORT

Information on patent family members

International Application No
PCT/US 01/20905

Patent document cited in search report		Publication date	Patent family member(s)			Publication date
US 6064810	A	16-05-2000	AU	4598497	A	17-04-1998
			EP	0928456	A1	14-07-1999
			JP	2001501336	T	30-01-2001
			WO	9813775	A1	02-04-1998
US 5255212	A	19-10-1993	JP	3276071	A	06-12-1991
EP 0709789	A	01-05-1996	JP	3034433	B2	17-04-2000
			JP	8128927	A	21-05-1996
			DE	69522407	D1	04-10-2001
			DE	69522407	T2	13-12-2001
			EP	0709789	A2	01-05-1996
			US	5699284	A	16-12-1997
WO 9940532	A	12-08-1999	AU	2761399	A	23-08-1999
			EP	1082691	A1	14-03-2001
			NO	20003947	A	04-08-2000
			WO	9940532	A1	12-08-1999
			US	6052520	A	18-04-2000